# An Approach to Planning Experiments for Life Data

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## Outline of Today's Talk

**I. Background**

**II. Assumptions, Models, and Basic Approach**

**III. Example**

**IV. Conclusions**

## I. Background.

Some people view reliability as:

*Quality over time.*

Industrial statisticians have been planning experiments for years to improve product quality and process performance.

Less work has gone into how to plan experiments to impact the reliability of the product and/or process.

Two major issues:

- The experts in DOE rarely know much about reliability data, which are very different from the data in most DOEs.
- The experts in reliability rarely know a great deal about DOE, especially the impact of experimental protocol on the model.

This talk represents a first, primitive step to bringing these two worlds together.

## II. Assumptions, Models, and Approach to the Analysis

**Two very basic concepts in DOE:**

- **experimental unit**

  **The smallest unit to which we apply the treatment combinations.**

- **observational unit.**

  **The actual unit measured.**

**The variability among the experimental units provides the basis for the experimental error.**

**The experimental error is the appropriate basis for all inference involving the experimental factors.**

**The variability among the observational units is part, *but only part!*, of the experimental error.**

**Consider a temperature - humidity chamber.**

**The chamber holds $n$ units.**

**If temperature and humidity are factors, then what are the experimental unit and the observational unit?**

**Impacts of confusing the experimental and observation units:**

- **If temperature and humidity are factors, then we overstate the appropriate degrees of freedom, and we use the too small an error term for inference.**
- **If temperature and humidity are purely accelerators, not factors, then the error from the experimental units is transmitted to the parameter estimates of the life distribution.**

  **The experimental error impacts the parameter estimates, which then impact the predicted performance at the use conditions.**

**Our assumptions:**

- **We place $n$ items on a test stand.**
  - **The test stand is the experimental unit.**
  - **The individual test items are the observational units.**
- **The life times for the individual units follow a Weibull distribution.**
- **The failure mechanism for each test combination is the same.**
- **Each test condition produces enough failures that the estimate of the scale parameter approximately follows a normal distribution.**

**If the life times follow a normal distribution, then we have a standard sub-sampling problem.**

**If we have $n$ items within each test stand, then an appropriate analysis uses only the sample means from each test stand as the response.**

**Problem: The life times do not follow a normal distribution!**

**The model within an experimental unit:**

**Let $t_{ij}$ be the observed life time for the $j^{th}$ item within the $i^{th}$ test stand.**

**For a Weibull distribution,**

$$f(t_{ij}) = \frac{\beta}{\eta_i}(\frac{t}{\eta_i})^{\beta-1}e^{-(\frac{t_{ij}}{\eta_i})^{\beta}} \quad t_{ij} > 0 \;\; \beta > 0 \;\; \eta_i > 0,$$

**where**

- $\beta > 0$ **is the shape parameter and**
- $\eta_i > 0$ **is the scale parameter.**

**It can be shown that if $t_{ij}$ follows a Weibull distribution, then $y_{ij} = \mathbf{ln}(t_{ij})$ follows the smallest extreme value distribution with pdf**

$$f(y_{ij}) = \frac{\beta}{\eta_i}e^{\frac{(y-\mu_i)}{\sigma}} \exp[-e^{\frac{(y-\mu_i)}{\sigma}}]$$

**where $\mu_i = \ln(\eta_i)$ and $\sigma = \beta^{-1}$.**

**Model for the experimental units:**

$$\hat{\eta}_i = \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i.$$

**where**

- $\epsilon_i \sim n(0, \sigma_\epsilon^2)$**.**
- $\hat{\eta}_i$ **is the MLE.**

**Basic idea is to do the usual DOE with $\hat{\eta}$, *which is asymptotically normal*, as the response!**

**In the process, we begin to account for the experimental unit variability.**

**The benefit to practitioners of this approach:**

**We can do everything in Minitab!**

**In the process, we take into account the true experimental variability!**

## III. Example

**Zellen (*Technometrics* 1959) performed an experiment looking at the impact of temperature and voltage on the life times of glass capacitors.**

**Meeker and Escobar (1998) analyze these data.**

**The results from Minitab estimating the $\eta$'s assuming a constant $\beta$.**

$$\hat{\beta} = 3.62$$

| volt | temp | $\hat{\eta}$ |
|---|---|---|
| 200 | 170 | 1262.35 |
| 200 | 180 | 1292.78 |
| 250 | 170 | 1207.58 |
| 250 | 180 | 532.85 |
| 300 | 170 | 683.61 |
| 300 | 180 | 431.04 |
| 350 | 170 | 633.86 |
| 350 | 180 | 510.10 |

**A problem test stand is:**

- **volt = 250**
- **temp = 180**

## Analysis from Minitab (and from Meeker and Escobar)

Regression Table

| Predictor | Coef | Standard Error | Z | P | 95.0% Normal CI Lower | Upper |
|---|---|---|---|---|---|---|
| Intercept | 13.4070 | 2.29584 | 5.84 | 0.000 | 8.90726 | 17.9068 |
| Voltage | -0.0059108 | 0.0010398 | -5.68 | 0.000 | -0.0079488 | -0.0038729 |
| Temperature | -0.0289047 | 0.0128970 | -2.24 | 0.025 | -0.0541822 | -0.0036271 |
| Shape | 2.74869 | 0.418739 | | | 2.03917 | 3.70509 |

Log-Likelihood = -244.242

## Note: This analysis models the $\mu = \ln(\eta)$'s, not the $\eta$'s.

## Note: The estimate of the shape parameter is not 3.62!

## Analysis Taking the Experimental Error into Consideration

```
The regression equation is
eta = 6621 - 4.86 volt - 25.5 temp
```

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 6621 | 2692 | 2.46 | 0.057 |
| volt | -4.859 | 1.362 | -3.57 | 0.016 |
| temp | -25.52 | 15.23 | -1.68 | 0.155 |

S = 215.366   R-Sq = 75.7%   R-Sq(adj) = 65.9%

## The results modeling the $\mu$'s are similar.

**Point Predictions:**

**New Analysis**

| volt | temp | $t_{.01}$ | $t_{.05}$ | $t_{.10}$ | $t_{.50}$ |
|---|---|---|---|---|---|
| **200** | **170** | **366.9** | **575.5** | **702.1** | **1181.4** |
| **200** | **180** | **297.0** | **465.9** | **568.3** | **956.3** |
| **250** | **170** | **299.1** | **469.3** | **572.5** | **963.3** |
| **250** | **180** | **228.1** | **357.8** | **436.5** | **734.5** |
| **300** | **170** | **231.4** | **363.0** | **442.9** | **745.2** |
| **300** | **180** | **159.2** | **249.8** | **304.7** | **512.8** |
| **350** | **170** | **163.7** | **256.8** | **313.2** | **527.1** |
| **350** | **180** | **90.4** | **141.8** | **172.9** | **291.09** |

**Traditional Analysis**

| volt | temp | $t_{.01}$ | $t_{.05}$ | $t_{.10}$ | $t_{.50}$ |
|---|---|---|---|---|---|
| **200** | **170** | **271.6** | **490.4** | **636.6** | **1260.0** |
| **200** | **180** | **216.5** | **390.9** | **507.4** | **1004.2** |
| **250** | **170** | **207.3** | **374.2** | **485.7** | **961.4** |
| **250** | **180** | **158.6** | **286.2** | **371.5** | **735.4** |
| **300** | **170** | **158.2** | **285.5** | **370.7** | **733.6** |
| **300** | **180** | **116.1** | **209.6** | **272.1** | **538.5** |
| **350** | **170** | **120.7** | **217.9** | **282.8** | **559.8** |
| **350** | **180** | **85.0** | **153.5** | **199.3** | **394.4** |

## IV. Conclusions

- This talk has taken only a first step to combining sound experimental design strategies for reliability data.
- Taking into account the experimental error has a non-trivial impact on the analysis.
- Need to develop confidence intervals for the predicted values.
  - Not a trivial issue: need to develop the appropriate likelihood.
  - Still requires the delta method.
  - Expect the new confidence intervals to be wider because of the experimental variability.
- Even then, the proposed analysis is naive.
- However, Minitab, or your other favorite software, can do the analysis.