Defect Initiation, Growth, and Failure – A General Statistical Model and Data Analyses

Wayne B. Nelson
Consultant, Schenectady, NY,
WNconsult@aol.com

Purpose: To present unpublished models and data analyses for defect initiation, growth, and failure.

OVERVIEW

APPLICATIONS

CIRCUIT BOARD DATA

BASIC STATISTICAL MODEL

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APPLICATIONS

Blisters in automotive paint.

Dendrites in circuit boards.

Cracks in jet engine components.

Cracks in catalytic converter containers.

Tumors in people.

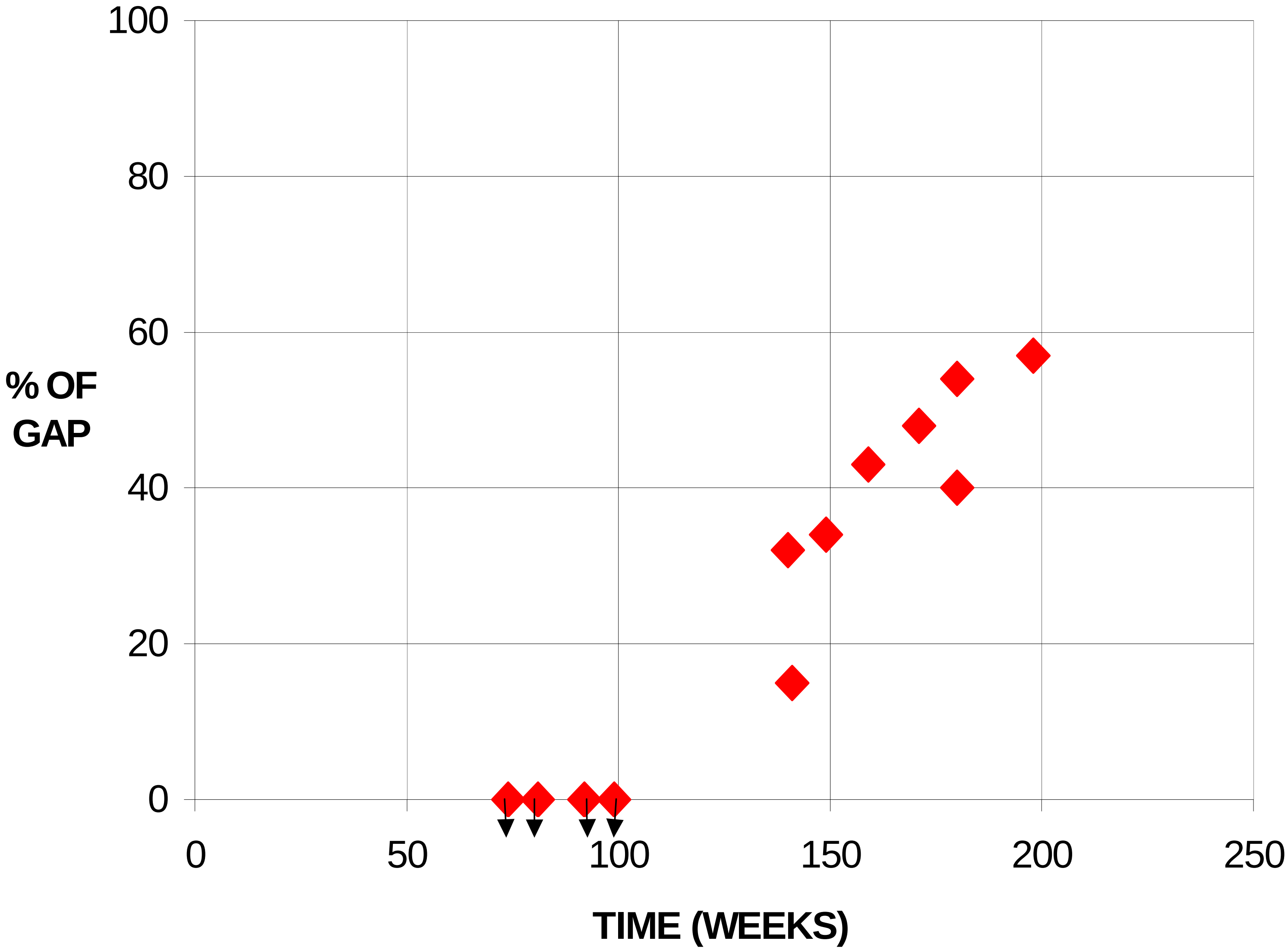
CIRCUIT BOARD DATA

Dendrites (copper filaments) growing between two parallel copper strips caused failures of hermetic circuit boards. Circuits of different ages were opened, and the longest dendrite in each was measured just once as a % of the distance grown between the strips. 0% indicates that no dendrite had yet initiated. My client wanted an estimate of the circuit life distribution.

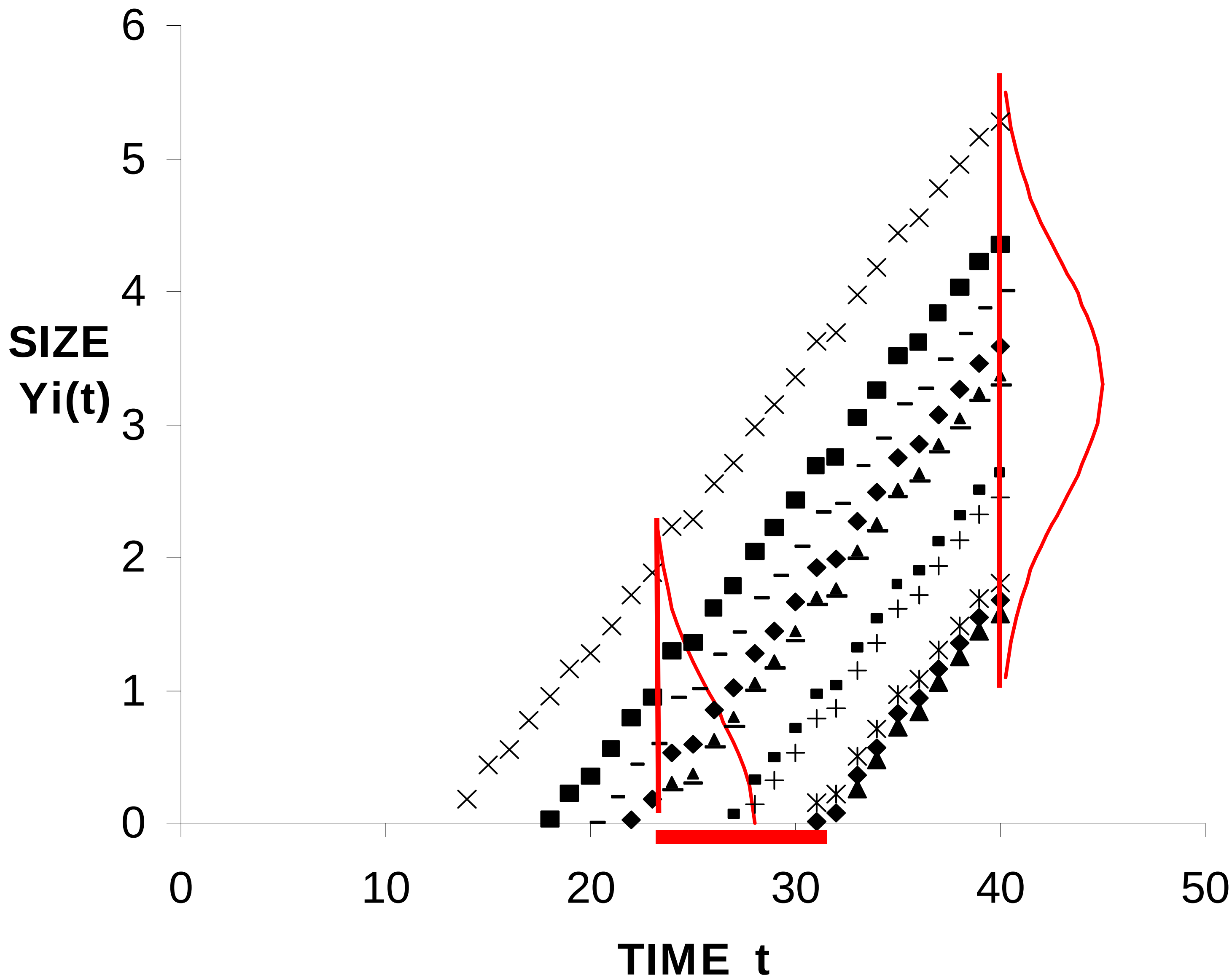
Table 1. Dendrite Age and Size Data

Circuit	Age Weeks	Dendrite Size %
1	171	48
2	81	0
3	74	0
4	180	54
5	198	57
6	149	34
7	180	40
8	159	43
9	92	0
10	141	15
11	140	32
12	99	0

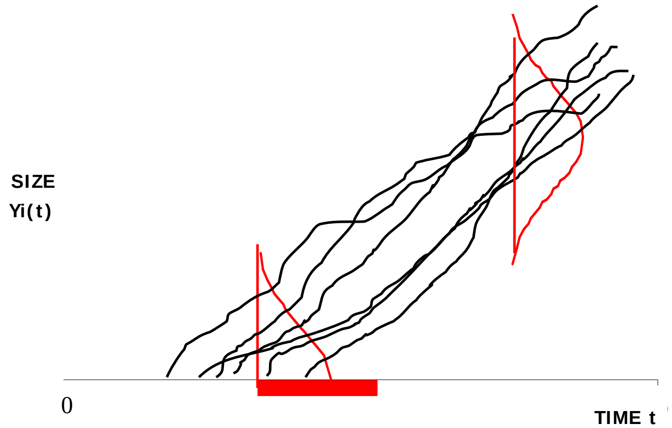
Figure 1. Display of dendrite size data.



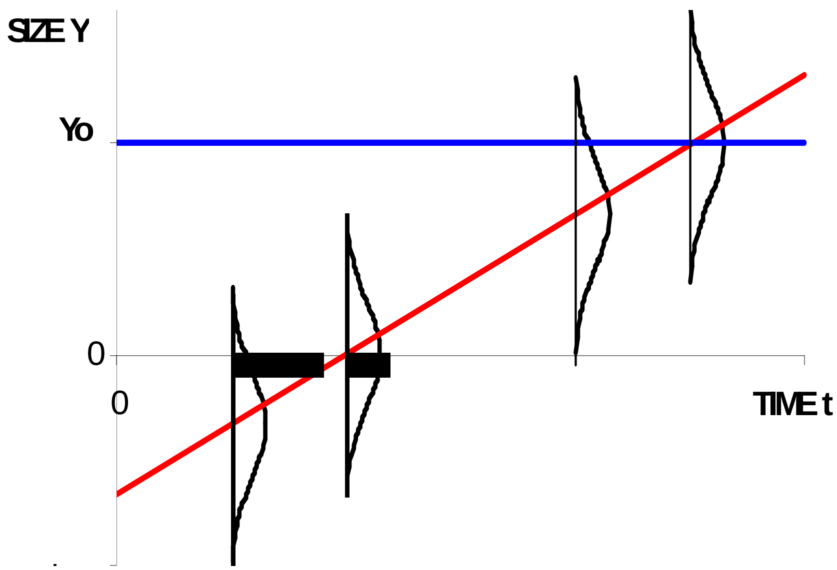
MOTIVATE MODEL – PAINT BLISTER SIZE (DIA.)



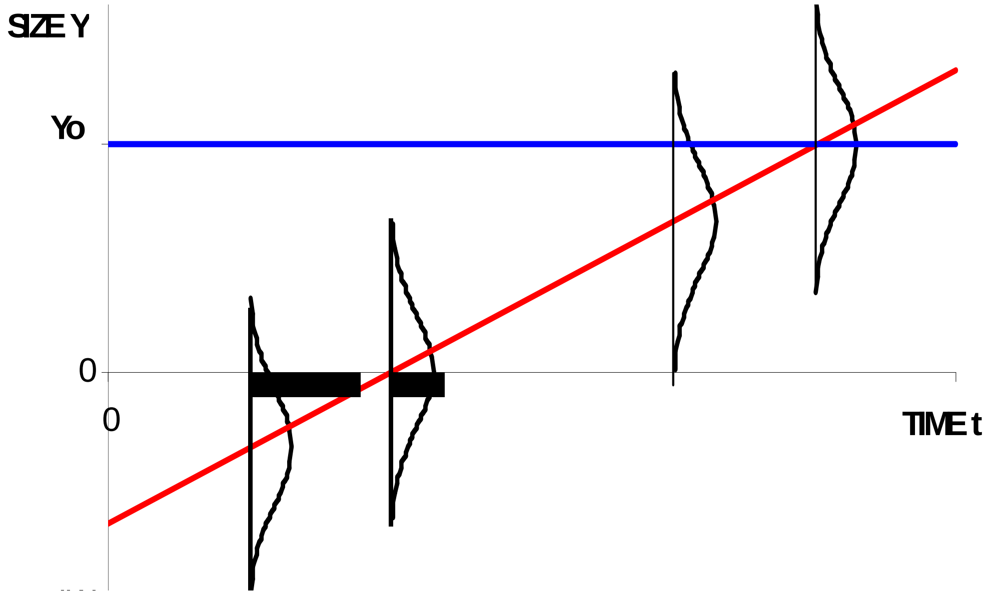
MOTIVATION – PRESUMED DENDRITE SIZE GROWTH



MODEL FOR SIZE VERSUS TIME:



BASIC STATISTICAL MODEL:

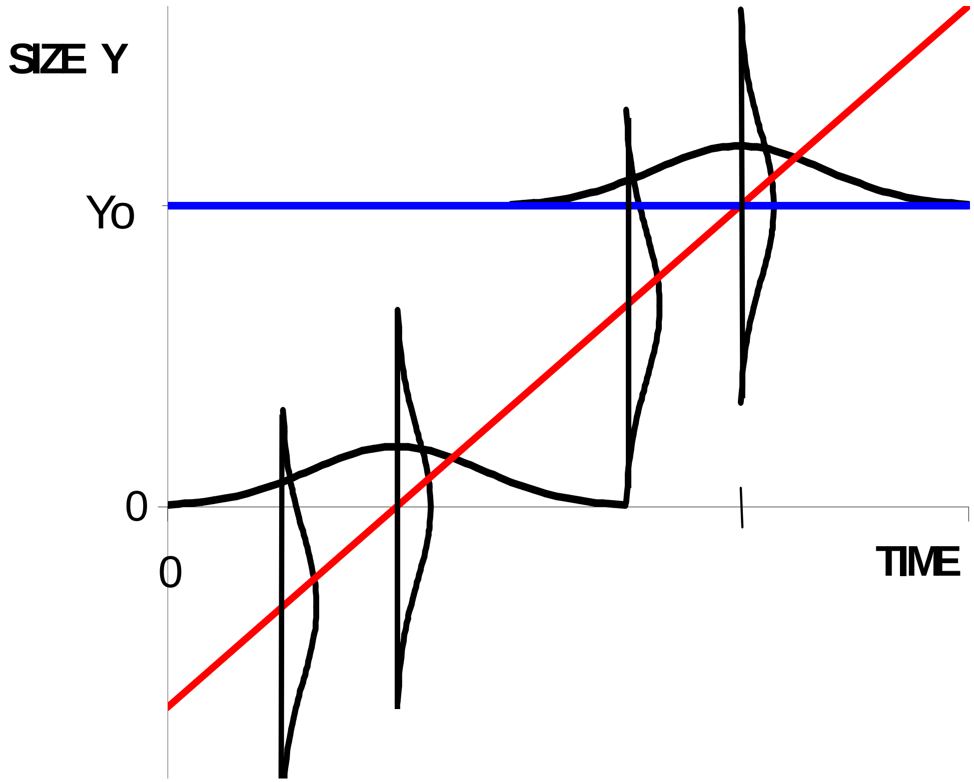


The cdf $F(y;t)$ of size Y at age t is normal with $\mu(t) = \gamma_0+\gamma_1 t$ and standard deviation σ, a simple linear-normal regression model,

$$F(y;t) = \Phi\{[y-\mu(t)]/\sigma\}.$$

This model lacks autocorrelation of size paths, as it cannot be estimated with just one measurement per specimen. Nelson (2004) and Meeker and Escobar (1998) present degradation models that lack a defect-free time to initiation.

DISTRIBUTION OF TIME TO INITIATION



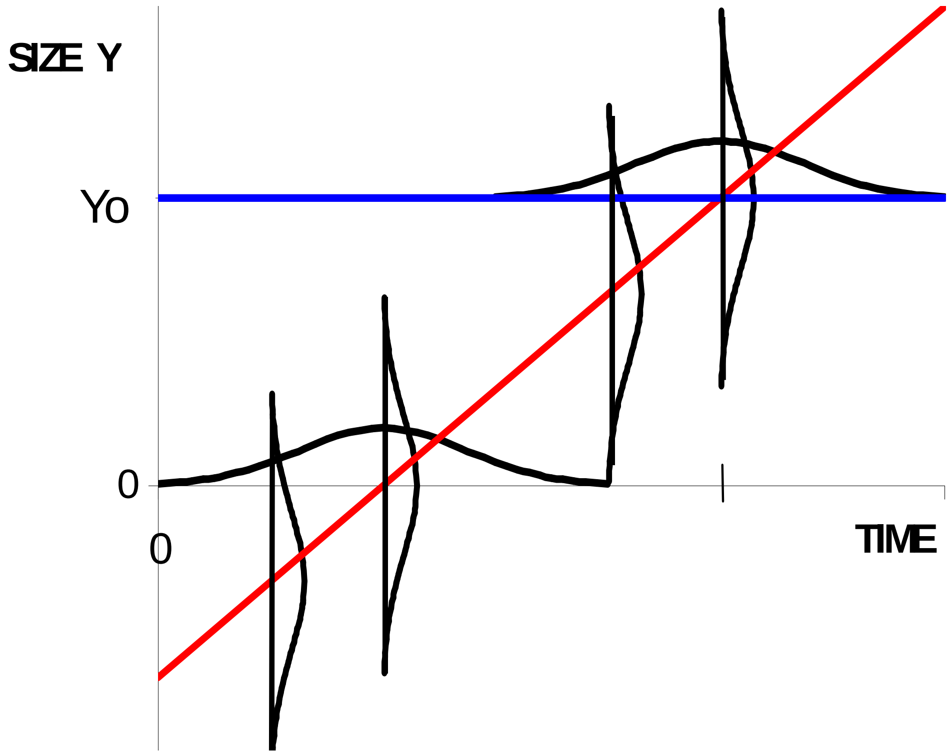
The population fraction that has initiated a defect by time t is

$G(t) = 1 – F(0;t) = 1 – \Phi\{[0 – \mu(t)]/\sigma\} = \Phi\{[t – (–\gamma_0/\gamma_1)]/(\sigma/\gamma_1)\}$.

This is normal with mean and standard deviation

$\mu_G = –\gamma_0/\gamma_1$ and $\sigma_G = \sigma/\gamma_1$.

DISTRIBUTION OF TIME TO FAILURE



The population fraction that has failed (size y_0) by time t is

$H(t) = 1 – F(y_0;t) = 1 – \Phi\{[\, y_0 – \mu(t)]/\sigma\} = \Phi\{[t – ((\, y_0 – \gamma_0)/\gamma_1))]/(\sigma/\gamma_1)\}$.

This is normal with mean and standard deviation

$\mu_G = (\, y_0 – \gamma_0)/\gamma_1$ and $\sigma_G = \sigma/\gamma_1$.

MODEL FITTING AND ANALYSES

The (size,age) data on the r (a random number) specimens with a defect are (Y_1,t_1), (Y_2,t_2), $\cdots$, (Y_r,t_r), where is the random number of units with a defect. Size data on the $n - r$ without a defect are left censored: $(0^-,t_{r+1})$, $(0^-,t_{r+2})$, $\cdots$, $(0^-,t_n)$.

Nelson (2004) describes various software that fit the simple linear-normal regression model (and others) to such data, using the method of maximum likelihood. Output includes estimates of model parameters and functions of them with approximate confidence limits.

DENDRITE MODEL FIT (Minitab)

Parameter	Estimate	95% Confidence Limits	
		Lower	Upper
γ_0 (weeks)	–69.05	–95.39	–42.70
γ_1 (%/week)	0.6605	0.4971	0.8240
σ (%)	6.651	4.073	10.86
μ_G (weeks)	104.5	*	*
σ_G (weeks)	10.1	*	*
μ_H (weeks)	255.9	*	*
σ_H (weeks)	10.1	*	*

* Minitab does not provide these limits. Nelson (2004) gives theory for such limits.

EXTENSIONS (WORK IN PROGRESS)

- Other functions, e.g., $\mu(t) = \gamma_0 + \gamma_1 \ln(t)$, $\sigma(t) = \exp(\gamma_0 + \gamma_1 t)$, and Paris' eq. for crack growth in fatigue.
- Other distributions, e.g., if $F(y;t)$ is largest extreme value and $\mu(t) = \gamma_0 + \gamma_1 \ln(t)$, $G(t)$ and $H(t)$ are Weibull.
- If measurements of size have random error, that needs to be included in the model.
- Analyses of residuals to assess the model distribution and data.
- Hypothesis tests to assess the adequacy of the model relationships.
- Best choice of ages $t_1, t_2, \cdots, t_n$ of sample specimens.

CONCLUDING REMAR KS

The basic model and ML fitting are a useful first step in the analysis of such data with defection initiation. They are presented in Nelson (2009).

Suitable physical models – distributions and relationships $\mu(t)$ and $\sigma(t)$ – need to be developed for specific applications.

REFERENCES

Meeker, W.Q. and Escobar, L.A. (1998), *Statistical Methods for Reliability Data*, Wiley, New York, www.wiley.com.

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