Alternatives to Resolution IV Screening Designs in 16 Runs

Bradley Jones
Director of Research & Development
JMP Division
SAS Institute

Douglas C. Montgomery
Arizona State University

Why are Regular Fractional Factorial Designs Useful for Factor Screening?

- The **sparsity of effects** principle
  - There may be lots of factors, but few are important
  - System is dominated by main effects, low-order interactions
- The **projection** property
  - Every fractional factorial contains full factorials in fewer factors
- **Sequential** experimentation
  - Can add runs to a fractional factorial to resolve difficulties (or ambiguities) in interpretation

The Regular One-Half Fraction of the 2^k

Montgomery, *Design & Analysis of Experiments,* 7th Edition, Wiley 2009

Notation: because the design has $2^k/2$ runs, it's referred to as a 2^{k-1}

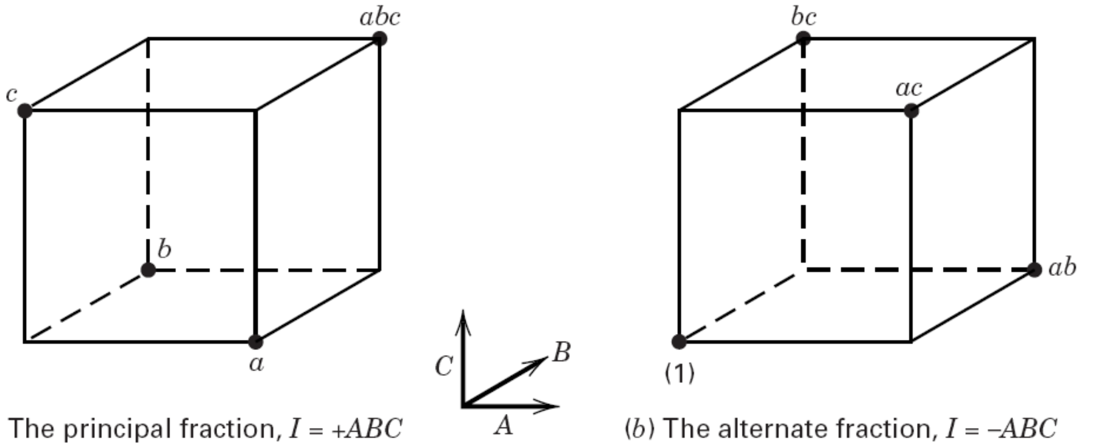
Consider a really simple case, the 2^{3-1} Note that $I = ABC$

■ **TABLE 8.1**

Plus and Minus Signs for the 2^3 Factorial Design

Treatment Combination	Factorial Effect							
	I	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
c	+	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	−	+	−	−	−
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−
(1)	+	−	−	−	+	+	+	−

The Regular One-Half Fraction of the 2^3



(a) The principal fraction, $I = +ABC$ (b) The alternate fraction, $I = -ABC$

■ **FIGURE 8.1** **The two one-half fractions of the 2^3 design**

For the principal fraction, notice that the contrast for estimating the main effect *A* is exactly the same as the contrast used for estimating the *BC* interaction.

This phenomena is called **aliasing** and it occurs in all fractional designs

Aliases can be found directly from the columns in the table of + and - signs

Aliasing in the One-Half Fraction of the 2^3

$A = BC$, $B = AC$, $C = AB$ (or $me = 2fi$)

Aliases can be found from the **defining relation** $I = ABC$ by multiplication:

$$AI = A(ABC) = A^2BC = BC$$

$$BI = B(ABC) = AC$$

$$CI = C(ABC) = AB$$

Textbook notation for aliased effects:

$$[A] \rightarrow A + BC, \quad [B] \rightarrow B + AC, \quad [C] \rightarrow C + AB$$

Construction of a Regular One-half Fraction

The **basic** design; the design **generator**

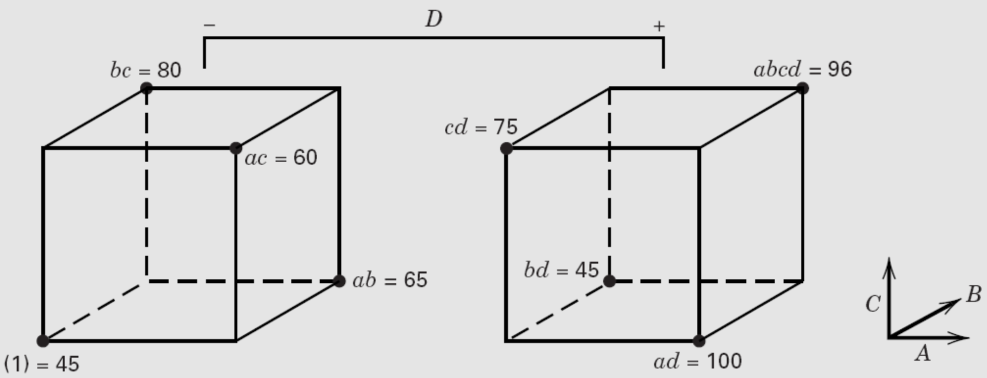
■ TABLE 8.2
The Two One-Half Fractions of the 2^3 Design

	Full 2^2 Factorial (Basic Design)		2^{3-1}_{III}, $I = ABC$			2^{3-1}_{III}, $I = -ABC$		
Run	A	B	A	B	$C = AB$	A	B	$C = -AB$
1	−	−	−	−	+	−	−	−
2	+	−	+	−	−	+	−	+
3	−	+	−	+	−	−	+	+
4	+	+	+	+	+	+	+	−

■ **TABLE 8.3**

The 2^{4-1}_{IV} Design with the Defining Relation I = ABCD

Run	Basic Design *A*	*B*	*C*	*D* = *ABC*	Treatment Combination	Filtration Rate
1	−	−	−	−	(1)	45
2	+	−	−	+	*ad*	100
3	−	+	−	+	*bd*	45
4	+	+	−	−	*ab*	65
5	−	−	+	+	*cd*	75
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	+	*abcd*	96



■ **FIGURE 8.3** The 2^{4-1}_{IV} design for the filtration rate experiment of Example 8.1

Interpretation of results relies on making some assumptions

e.g. **Effect Heirarchy**

Confirmation experiments are essential

Adding more runs to resolve ambiguities

■ **TABLE 8.4**

Estimates of Effects and Aliases from Example 8.1[a]

Estimate	Alias Structure
[A] = 19.00	$[A] \rightarrow A + BCD$
[B] = 1.50	$[B] \rightarrow B + ACD$
[C] = 14.00	$[C] \rightarrow C + ABD$
[D] = 16.50	$[D] \rightarrow D + ABC$
[AB] = −1.00	$[AB] \rightarrow AB + CD$
[AC] = −18.50	$[AC] \rightarrow AC + BD$
[AD] = 19.00	$[AD] \rightarrow AD + BC$

[a] Significant effects are shown in boldface type.

A Priori and Aliasing Models

Consider two kinds of effects:

Primary effects are ones you are sure you want to estimate. There are p_1 of these.

Potential effects are ones you are afraid to ignore. There are p_2 of these.

The specified number of runs, n, is often less than the number of primary and potential effects.

$$p_1 < n < p_1 + p_2$$

Alias Matrix Definition

Suppose we believe in the model:

$y = X_1\beta_1 + \varepsilon$

But the true model is:

$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$

Then, the Alias matrix is:

$A = (X_1^t X_1)^{-1} X_1^t X_2$

The model for the example on the next slide is:

$y = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_3\beta_3 + \varepsilon$

In the notation defined above.

$$\boldsymbol{\beta}_1 = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad \text{and} \quad \mathbf{X}_1 = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Suppose that the true model contains all the two-factor interactions, so that

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \epsilon$$

and

$$\boldsymbol{\beta}_2 = \begin{bmatrix} \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{bmatrix}, \quad \text{and} \quad \mathbf{X}_2 = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Now

$$\mathbf{X}_1'\mathbf{X}_1 = 4\,\mathbf{I}_4 \quad \text{and} \quad \mathbf{X}_1'\mathbf{X}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

Therefore,

$$(\mathbf{X}_1'\mathbf{X}_1)^{-1} = \frac{1}{4}\mathbf{I}_4$$

and

$$E(\hat{\boldsymbol{\beta}}_1) = \boldsymbol{\beta}_1 + \mathbf{A}\,\boldsymbol{\beta}_2$$

$$E\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\\ \hat{\beta}_2\\ \hat{\beta}_3\end{bmatrix} = \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{bmatrix} + \frac{1}{4}\mathbf{I}_4\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 4\\ 0 & 4 & 0\\ 4 & 0 & 0\end{bmatrix}\begin{bmatrix}\beta_{12}\\ \beta_{13}\\ \beta_{23}\end{bmatrix}$$

$$= \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\beta_{12}\\ \beta_{13}\\ \beta_{23}\end{bmatrix}$$

$$= \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{bmatrix} + \begin{bmatrix}0\\ \beta_{23}\\ \beta_{13}\\ \beta_{12}\end{bmatrix}$$

$$= \begin{bmatrix}\beta_0\\ \beta_1 + \beta_{23}\\ \beta_2 + \beta_{13}\\ \beta_3 + \beta_{12}\end{bmatrix}$$

Main effects aliased with the 2fis

The Alias Matrix:

Columns contain the aliasing model terms.

Rows contain the parameters in the model that you fit.

Effect	A*B	A*C	B*C
Intercept	0	0	0
A	0	0	1
B	0	1	0
C	1	0	0

The 2^{6-2} Fractional-Factorial Design

■ **TABLE 8.9**

Construction of the 2_{IV}^{6-2} Design with the Generators $I = ABCE$ and $I = BCDF$

	Basic Design					
Run	A	B	C	D	$E = ABC$	$F = BCD$
1	−	−	−	−	−	−
2	+	−	−	−	+	−
3	−	+	−	−	+	+
4	+	+	−	−	−	+
5	−	−	+	−	+	+
6	+	−	+	−	−	+
7	−	+	+	−	−	−
8	+	+	+	−	+	−
9	−	−	−	+	−	+
10	+	−	−	+	+	+
11	−	+	−	+	+	−
12	+	+	−	+	−	−
13	−	−	+	+	+	−
14	+	−	+	+	−	−
15	−	+	+	+	−	+
16	+	+	+	+	+	+

Aliasing of the 2^{6-2} Fractional-Factorial Design

Complete defining relation: $I = ABCE = BCDF = ADEF$

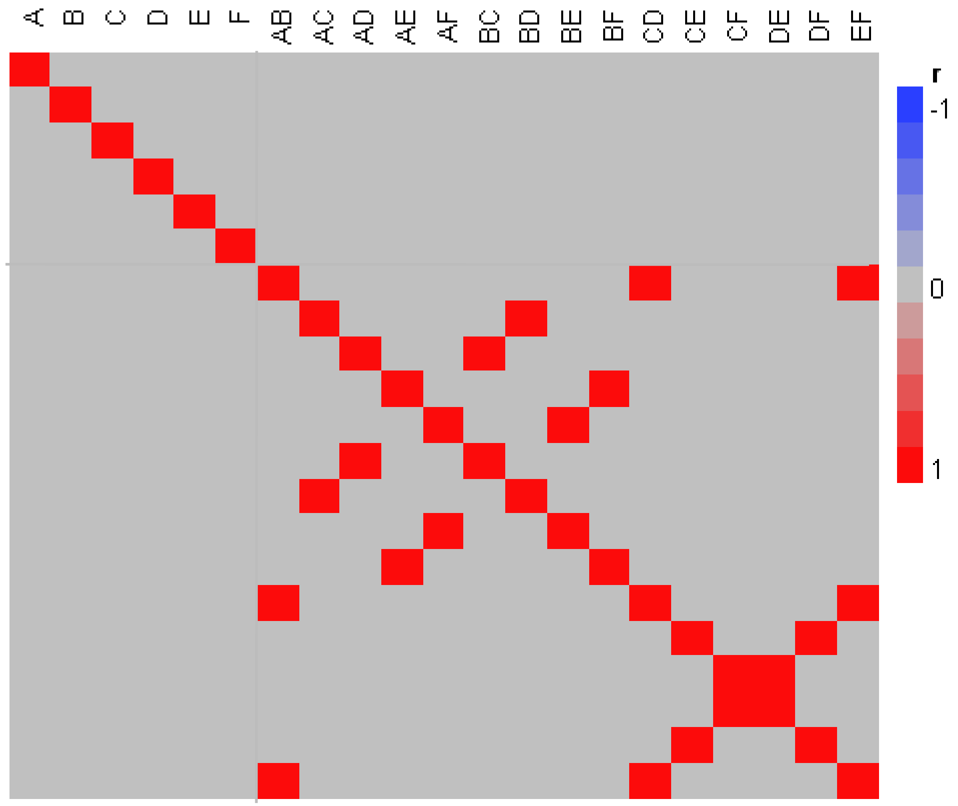
■ **TABLE 8.8**

Alias Structure for the 2_{IV}^{6-2} Design with $I = ABCE = BCDF = ADEF$

$A = BCE = DEF = ABCDF$	$AB = CE = ACDF = BDEF$
$B = ACE = CDF = ABDEF$	$AC = BE = ABDF = CDEF$
$C = ABE = BDF = ACDEF$	$AD = EF = BCDE = ABCF$
$D = BCF = AEF = ABCDE$	$AE = BC = DF = ABCDEF$
$E = ABC = ADF = BCDEF$	$AF = DE = BCEF = ABCD$
$F = BCD = ADE = ABCEF$	$BD = CF = ACDE = ABEF$
	$BF = CD = ACEF = ABDE$
$ABD = CDE = ACF = BEF$	
$ACD = BDE = ABF = CEF$	

This is a **regular** design

Color Plot of the Correlation Matrix for the Minimum Aberration Regular FF Design



Are Regular FF the Best Screening Designs?

- In regular designs the alias matrix consists of either 0, +1 or -1 entries
- That means that effects are completely confounded
- Unless the experimenter has some “process knowledge”, effects cannot be separated without conducting additional experiments
  - Fold-over
  - Partial fold-over
  - Optimal augmentation

Number of Orthogonal Designs versus Number of Factors

Number of Factors	Number of Nonisomorphic Designs
6	27
7	55
8	80

Define Nonisomorphic

Two designs are nonisomorphic if you cannot get one from the other by:

- Permuting rows
- Permuting columns
- Relabeling the level names

Designs that Avoid Pure Confounding

The 16-run minimum aberration resolution IV designs (6, 7, and 8 factors) are among the most widely used designs in practice.

There exist designs with no pure confounding that are superior to the standard minimum aberration resolution IV designs in the sense that they offer a better chance of detecting significant two-factor interactions.

These designs are constructed from the Hall orthogonal arrays.

Hall I 15 Factor Design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
2	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
3	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
8	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
9	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1
10	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
11	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
12	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
14	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
15	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Hall II 15 Factor Design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
16	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1

Hall III 15 Factor Design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1
16	-1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1

Hall IV 15 Factor Design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1
13	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1
16	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1

Hall V 15 Factor Design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	-1	1	-1	1	-1	1	-1
8	1	-1	-1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1
9	-1	1	-1	1	-1	1	-1	1	1	-1	-1	-1	-1	1	1
10	-1	1	-1	1	-1	1	-1	-1	-1	1	1	1	1	-1	-1
11	-1	1	-1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1
12	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1
13	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	1
14	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1
15	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
16	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1

Constructing the Recommended 6 Factor Design

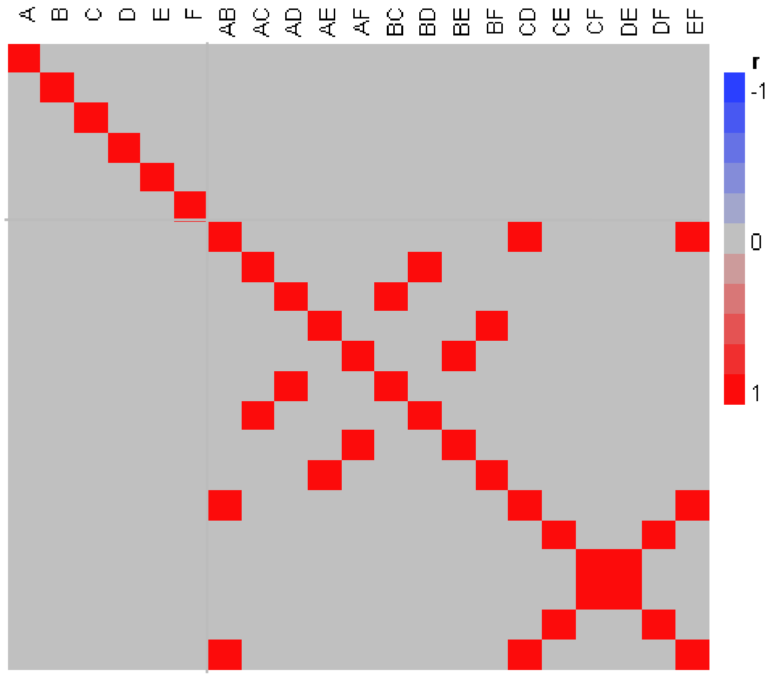
Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
16	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1

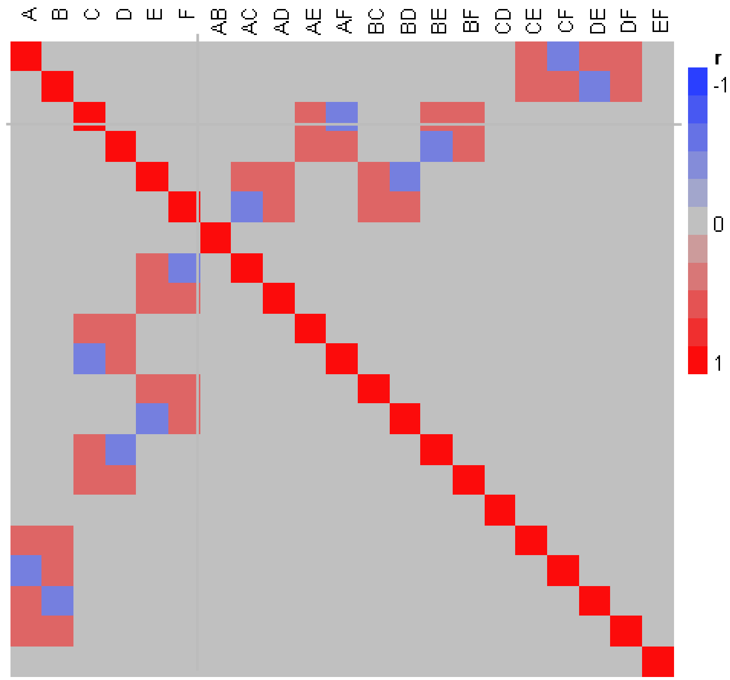
Hall II – Columns D, E, H, K, M, Q

Recommended Nonregular 6 Factor Design

Run	A	B	C	D	E	F
1	1	1	1	1	1	1
2	1	1	-1	-1	-1	-1
3	-1	-1	1	1	-1	-1
4	-1	-1	-1	-1	1	1
5	1	1	1	-1	1	-1
6	1	1	-1	1	-1	1
7	-1	-1	1	-1	-1	1
8	-1	-1	-1	1	1	-1
9	1	-1	1	1	1	-1
10	1	-1	-1	-1	-1	1
11	-1	1	1	1	-1	1
12	-1	1	-1	-1	1	-1
13	1	-1	1	-1	-1	-1
14	1	-1	-1	1	1	1
15	-1	1	1	-1	1	1
16	-1	1	-1	1	-1	-1

Comparison of Correlation Plots





The design without pure confounding is orthogonal for the main effects model.

No two-factor interactions are aliased with each other

Maximum absolute correlation is 0.5.

Photoresist Experiment (2^{6-2}) from Montgomery (2009)

Run	A	B	C	D	E	F	Thickness
1	-1	-1	-1	-1	-1	-1	4524
2	1	-1	-1	-1	1	-1	4657
3	-1	1	-1	-1	1	1	4293
4	1	1	-1	-1	-1	1	4516
5	-1	-1	1	-1	1	1	4508
6	1	-1	1	-1	-1	1	4432
7	-1	1	1	-1	-1	-1	4197
8	1	1	1	-1	1	-1	4515
9	-1	-1	-1	1	-1	1	4521
10	1	-1	-1	1	1	1	4610
11	-1	1	-1	1	1	-1	4295
12	1	1	-1	1	-1	-1	4560
13	-1	-1	1	1	1	-1	4487
14	1	-1	1	1	-1	-1	4585
15	-1	1	1	1	-1	1	4195
16	1	1	1	1	1	1	4510

Screening Analysis for the Photoresist Experiment

Term	Contrast	Lenth t-Ratio	p-Value	Aliases
A	85.3125	6.11	0.0007*	B*C*E, E*F*D
B	-77.6875	-5.56	0.0014*	A*C*E, C*F*D
C	-34.1875	-2.45	0.0309*	A*B*E, B*F*D
E	21.5625	1.54	0.1298	A*B*C, A*F*D
F	-14.6875	-1.05	0.2690	B*C*D, A*E*D
D	7.5625	0.54	0.6158	B*C*F, A*E*F
A*B	54.8125	3.92	0.0074*	C*E
A*C	-3.4375	-0.25	0.8175	B*E
B*C	3.3125	0.24	0.8242	A*E, F*D
A*F	-16.4375	-1.18	0.2231	E*D
B*F	8.0625	0.58	0.5900	C*D
C*F	-2.6875	-0.19	0.8563	B*D
E*F	10.5625	0.76	0.4166	A*D
A*B*F	10.8125	0.77	0.4065	C*E*F, A*C*D, B*E*D
A*C*F	-5.6875	-0.41	0.7028	B*E*F, A*B*D, C*E*D

Design Without Pure Confounding for the Photoresist Application Experiment

Run	A	B	C	D	E	F	Thickness
1	1	1	1	1	1	1	4494
2	1	1	-1	-1	-1	-1	4592
3	-1	-1	1	1	-1	-1	4357
4	-1	-1	-1	-1	1	1	4489
5	1	1	1	-1	1	-1	4513
6	1	1	-1	1	-1	1	4483
7	-1	-1	1	-1	-1	1	4288
8	-1	-1	-1	1	1	-1	4448
9	1	-1	1	1	1	-1	4691
10	1	-1	-1	-1	-1	1	4671
11	-1	1	1	1	-1	1	4219
12	-1	1	-1	-1	1	-1	4271
13	1	-1	1	-1	-1	-1	4530
14	1	-1	-1	1	1	1	4632
15	-1	1	1	-1	1	1	4337
16	-1	1	-1	1	-1	-1	4391

Where Did We Get the Data for this Experiment?

We simulated it.

We chose the significant main effects A, B, C, and E along with the interaction CE.

We selected the random component to have the same standard deviation as the original data

The result is data that represents closely the original experiment if the design without pure confounding had been run

Stepwise Analysis of Data from Design without Pure Confounding

Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
X	Intercept	4462.8125	1	0	0.000	1
X	A	85.3125	1	77634.37	53.976	2.46e-5
X	B	-77.6825	1	64368.76	44.753	5.43e-5
X	C	-34.1875	2	42735.84	14.856	0.00101
	D	0	1	31.19857	0.020	0.89184
X	E	21.5625	2	31474.34	10.941	0.00304
	F	0	1	2024.045	1.474	0.25562
	A*B	0	1	395.8518	0.255	0.6259
	A*C	0	1	476.1781	0.308	0.59234
	A*D	0	2	3601.749	1.336	0.31571
	A*E	0	1	119.4661	0.075	0.78986
	A*F	0	2	4961.283	2.106	0.18413
	B*C	0	1	60.91511	0.038	0.84923
	B*D	0	2	938.8809	0.279	0.76337
	B*E	0	1	3677.931	3.092	0.11254
	B*F	0	2	2044.119	0.663	0.54164
	C*D	0	2	1655.264	0.520	0.61321
X	C*E	54.8125	1	24035.28	16.711	0.00219
	C*F	0	2	2072.497	0.673	0.53667
	D*E	0	2	79.65054	0.022	0.97803
	D*F	0	0	0	.	.
	E*F	0	2	5511.275	2.485	0.14476

Constructing the Recommended 7 Factor Design

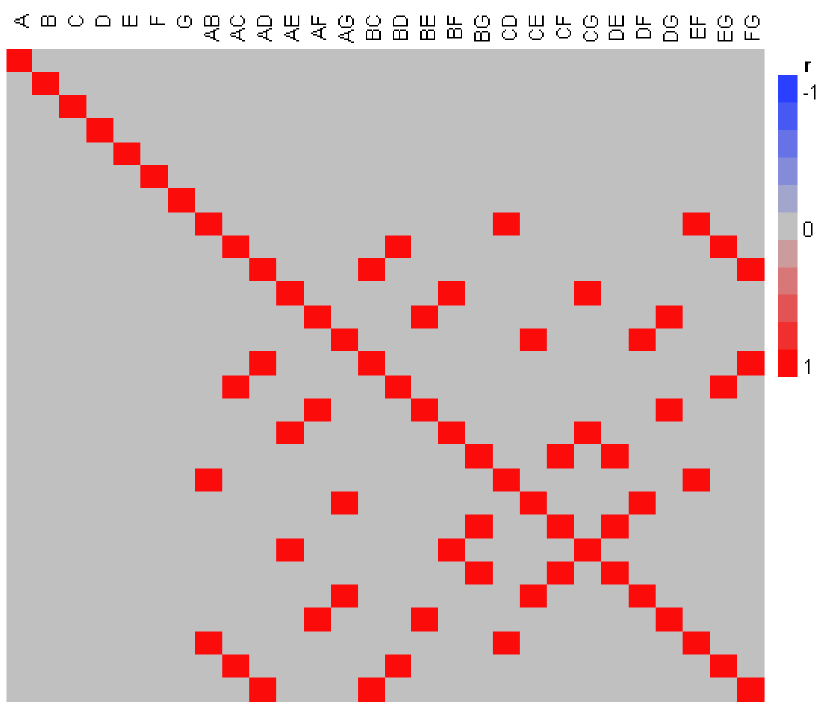
Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1
16	-1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1

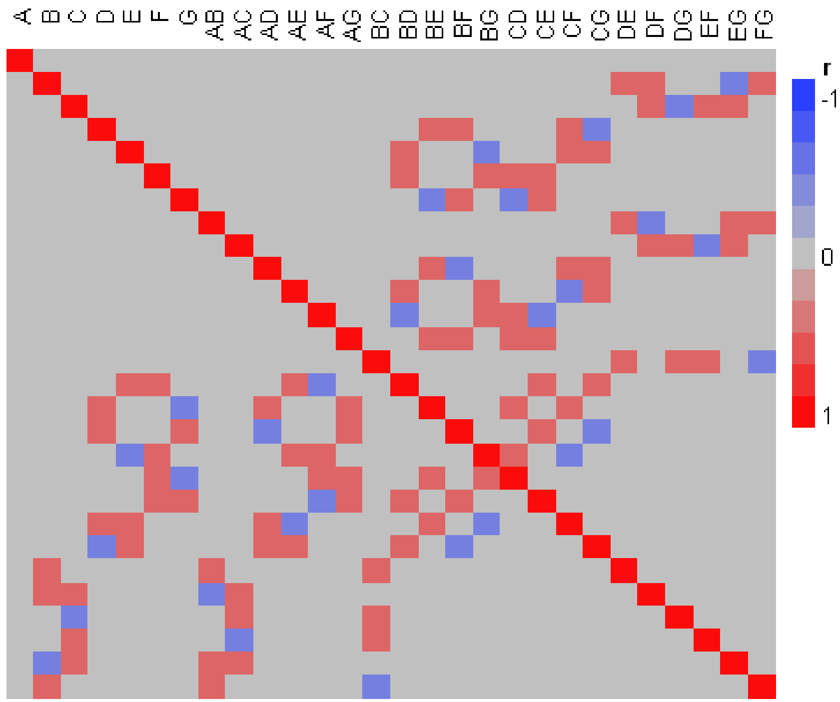
Hall III – Columns A, B, D, H, J, M, Q

Recommended Nonregular 7 Factor Design

Run	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	1	1	-1	-1	-1	-1
3	1	1	-1	1	1	-1	-1
4	1	1	-1	-1	-1	1	1
5	1	-1	1	1	-1	1	-1
6	1	-1	1	-1	1	-1	1
7	1	-1	-1	1	-1	-1	1
8	1	-1	-1	-1	1	1	-1
9	-1	1	1	1	1	1	-1
10	-1	1	1	-1	-1	-1	1
11	-1	1	-1	1	-1	1	1
12	-1	1	-1	-1	1	-1	-1
13	-1	-1	1	1	-1	-1	-1
14	-1	-1	1	-1	1	1	1
15	-1	-1	-1	1	1	-1	1
16	-1	-1	-1	-1	-1	1	-1

Correlation Plots for the Standard and No-confounding Designs





The recommended design is orthogonal and does not have any complete confounding of effects

Constructing the Recommended 8 Factor Design

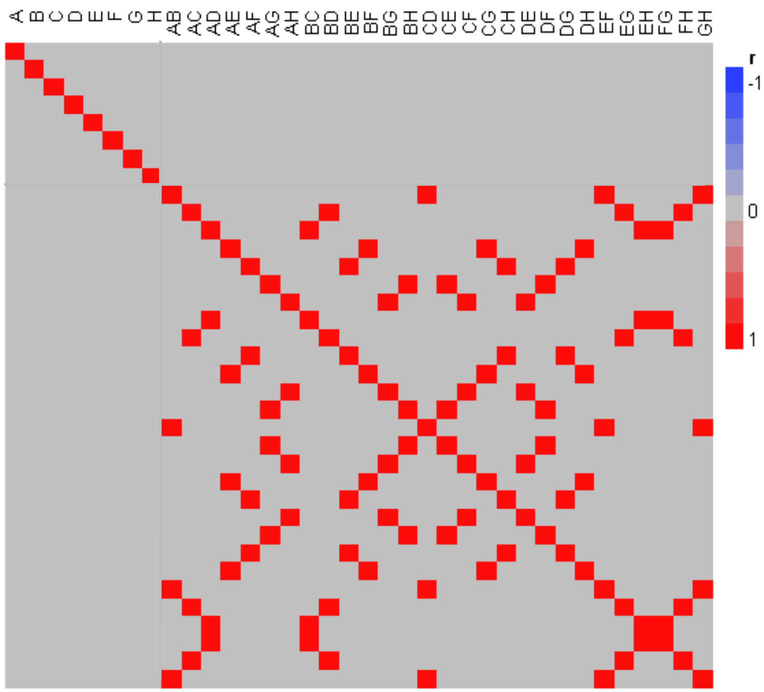
Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
3	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
9	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
10	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1
11	-1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
12	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1
13	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1
14	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
15	-1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1
16	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1

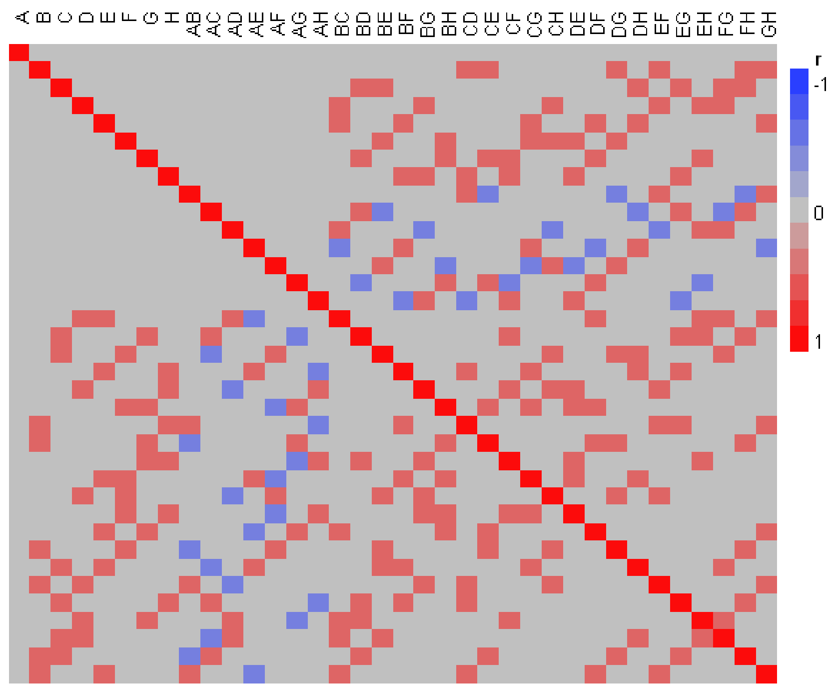
Hall IV – Columns A, B, D, F, H, J, M, P

Recommended Nonregular 8 Factor Design

Run	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1
2	1	1	1	1	-1	-1	-1	-1
3	1	1	-1	-1	1	1	-1	-1
4	1	1	-1	-1	-1	-1	1	1
5	1	-1	1	-1	1	-1	1	-1
6	1	-1	1	-1	-1	1	-1	1
7	1	-1	-1	1	1	-1	-1	1
8	1	-1	-1	1	-1	1	1	-1
9	-1	1	1	1	1	1	1	1
10	-1	1	1	-1	1	-1	-1	-1
11	-1	1	-1	1	-1	-1	1	-1
12	-1	1	-1	-1	-1	1	-1	1
13	-1	-1	1	1	-1	-1	-1	1
14	-1	-1	1	-1	-1	1	1	-1
15	-1	-1	-1	1	1	1	-1	-1
16	-1	-1	-1	-1	1	-1	1	1

Correlation Plots for the Standard and No-confounding Designs





The recommended design is orthogonal and does not have any complete confounding of effects

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