



Research & Technology

Applications of Reliability Demonstration Test

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Outline

- **Concept of Reliability Demonstration Test**
- **Connection between test outcome and test conditions**
- **Adjustment with one test variable**
- **Adjustment with two test variables**
- **Adjustment with three test variables**
- **General remarks**

Concept of RDT

- **Set a test plan such that successful outcome of this test plan would demonstrate reliability at R0 with confidence C0**
- **For example, if at most 1 failure occurs out of testing 46 items under certain test conditions, then the test has demonstrated 90% reliability with 95% confidence.**
- **Can one afford 46 items?**
 - **Cost constraints: financial, effort, time**
 - **Complex items are hard to come by.**
 - **Durable product may take a long time to acquire data**
 - **etc.**
- **References:**
 - **Lipson, C. and Sheth, N.J. (1973), *Statistical Design and Analysis of Engineering Experiments*, New York: McGraw Hill.**
 - **Martz, H.F. and Waller, R.A. (1982), *Bayesian Reliability Analysis*, New York: John Wiley and Sons.**
 - **Wang, C.J. (1991), "Sample Size Determination of Bogey Tests without Failure," *Quality and Reliability Engineering International*, 7, 35-38.**
 - **Doganaksoy, N. (1995), "Determining the Duration of a Demonstration Life Test Before All Units Fail," *IEEE Transaction on Reliability*, 44, 26-30.**
 - **Meeker, W.Q., Hahn, G. and Doganaksoy, N. (2004), "Statistics roundtable: Planning Life Tests for Reliability Demonstration," *Quality Progress*, 37, 80-82.**
 - **Mease, D. and Nair, V.N. (2006), "Extreme Testing with Binary Data and Application to Reliability Demonstration," *Technometrics*, 48, 399-410.**

Connecting between test outcome and test conditions

- **Definition of a failure:**
 - test article fails, breaks, changes from its normal state, passes a threshold, etc ...
- **Test conditions:**
 - static stress load, fatigue cycles with constant load, step-stress loading, mechanical stress and environmental stress, etc ...
- **Past knowledge:**
 - Test at normal conditions, test at higher “stress” conditions, simple specimen, small articles, lower portion of the building block.
 - Rigorously controlled tests (sometimes regulated)
 - Statistical models, physical and chemical models, computer models

Connecting between test outcome and test conditions

- **Binomial probability**

- at most k failures out of n tests provides a confidence interval for p
- Examples of k and n for 95% confidence

Demonstrated Reliability	Max. Failure k	Sample Size n
0.90	0	29
0.90	1	46
0.90	2	61
0.95	0	59
0.95	1	93
0.95	2	124
0.99	0	299
0.99	1	473
0.99	2	628

- **Knowledge of the relationship of p and test conditions enables planning RDT.**
 - **Example: the breaking threshold of a tempered glass specimen is modeled by a log-logistic distribution.**
 - model is the same as “Quantal Response” problem in drug dosage study
 - threshold is not a perfect step function
 - **Instead of testing n specimens at a pre-specified level, can one test m specimens and still demonstrate the same level of reliability and confidence?**

Test with one variable

- **Tempered glass strength test**
 - **Architectural use, large appliances, public vehicles, furniture**
 - **Quick quenching of heated glass creates an strong surface tension on flat glass. This surface tension is the key to tempered glass.**
 - **Difficult to break. When it breaks, it shatters into tiny pieces.**
 - **Typical assessment of strength is done by an impact test.**
- **To illustrate the strength of a new tempering process satisfying a certain level, one would illustrate it by impact testing n specimens and finding zero failure.**

Test with one variable

- **Based on impact tests on specimens from this new process, the log-logistic model provides a good description of the breaking threshold.**

$$p(x) = \frac{\exp(\beta_0 + \beta_1 \log(x))}{1 + \exp(\beta_0 + \beta_1 \log(x))}$$

- **The parameters are estimated to be** $\beta_0 = -8.7431$ $\beta_1 = 3.7971$
- **To demonstrate 90% reliability with 95% confidence, one would need to show zero failure in 29 tests, and tested at level 5.606**

$$x_0 = \exp\left\{\log\left(\frac{0.1}{0.9} - \beta_0\right) / \beta_1\right\} = 5.606$$

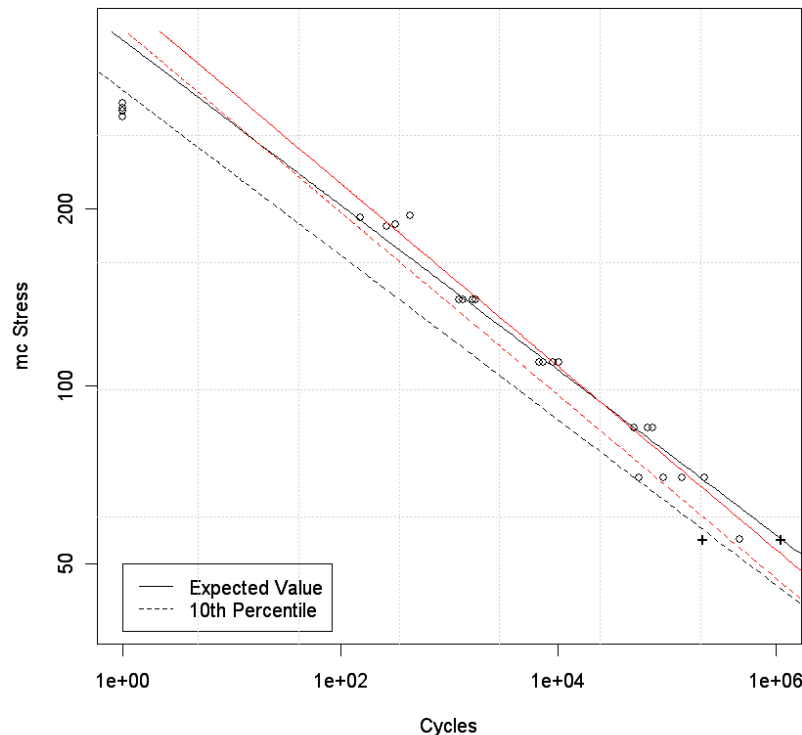
- **What if one would test only 10 specimens to demonstrate the same level of reliability and confidence?**

$$x_d = \exp\left\{\log\left(\frac{0.2589}{0.7411} - \beta_0\right) / \beta_1\right\} = 7.581$$

- **With a 35% increase in impact load, one could “get away” with less number of tests.**

Test with two variables

- **Fatigue life test on a part with constant load stress**
- **Multiple stress levels, right censored, life distribution**
- **Example: Sendeckyj (1981) S-glass epoxy**
 - Weibull regression model with log-link with log stress



$$R(y | x) = \exp \left[- \left(\frac{y}{\alpha(x)} \right)^\beta \right]$$

$$\log[\alpha(x)] = \theta_0 + \theta_1 \log(x)$$

Test with two variables

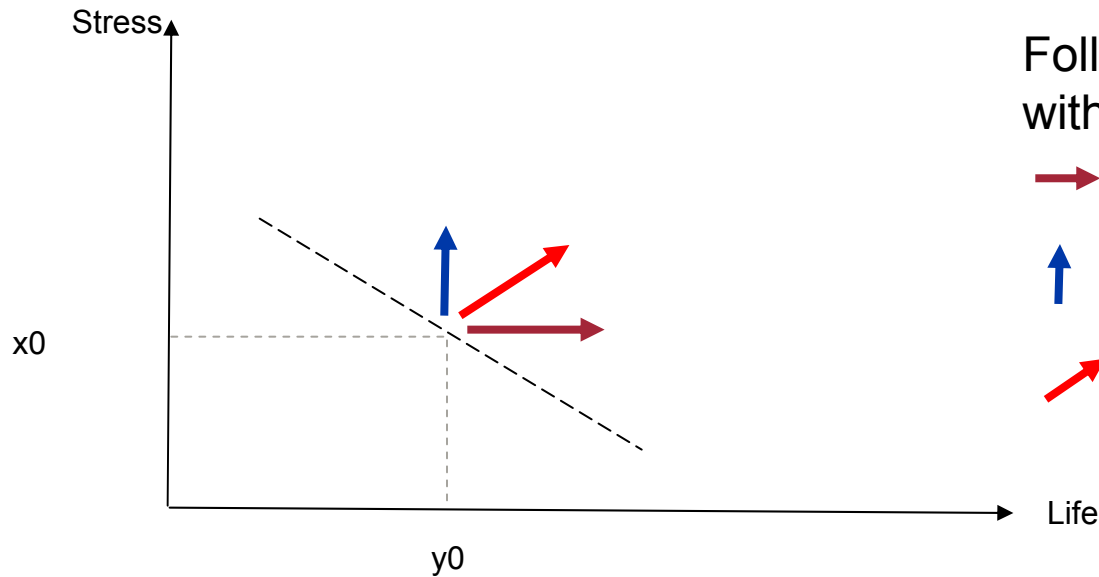
- **Failure requirements on life y_0 and stress x_0**
- **zero failure out of 29 tests would demonstrate 90% reliability with 95% confidence**
- **In very reliable products and very complex parts, the desire to obtain result early and to shorten development time drives accelerated tests.**
- **Example: estimates for the Sendeckyj data are**

$$\theta_0 = 39.4836 \quad \theta_1 = -6.4390 \quad \beta = 2.6730$$

- **To demonstrate $y_0=50,000$ cycles, $x_0=75.25$ with 90% reliability with 95% confidence, one needs zero failure among 29 tests.**
- **To demonstrate the same reliability and confidence with zero failures out of $n_d=5$ specimens, one would test the units at a higher stress $x_d=1.039*x_0$ until a longer life $y_d=1.5*50,000$.**

Test with two variables

- **Failure requirement: y_0 and x_0**
- **0 out of 29 for 90% reliability and 95% confidence**
- **What if 0 out of n_d tests at x_0 but run until $y_d.x$?**
- **What if 0 out of n_d tests at $x_d.y$ but run until y_0 ?**
- **What if 0 out of n_d tests at x_d and run until y_d ?**



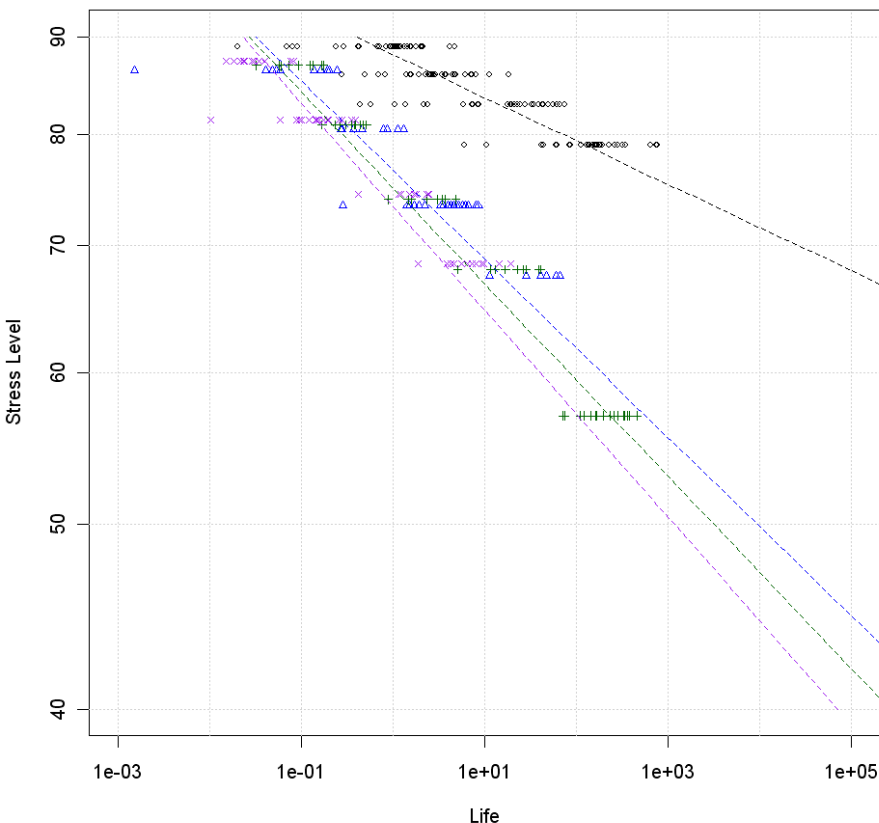
Follow the Sendeckyj example with $n_d=1$

- $y_d.x=3.499$
- ↑ $x_d.y=1.215$
- ↗ $x_d =2, y_d=1.091$

Test with three variables

- **Fatigue life test on a part with mechanical and environmental stress**
- **Failure requirements: y_0 life, x_0 stress, t_0 temperature**
- **Historical test data shows a log-Normal regression with stress and temperature affecting the life distribution**

Life Distribution at Different Temperatures



$$R(y | x, t) = 1 - \Phi\left(\frac{\log y - \mu(x, t)}{\sigma}\right)$$

$$\mu(x, t) = \beta_0 + \beta_1 \log(x) + \beta_2 \log(t) + \beta_{12} \log(x) * \log(t)$$

Test with three variables

- **Suppose that one would use 29 parts to demonstrate 90% reliability with 95% confidence.**
- **Requirement: zero out of 29 tested at $x_0 = 52.5$, $t_0 = 80$ and last until $y_0 = 10000$.**
- **To demonstrate the same reliability and with the same confidence with 10 parts, one could execute either one of the following plans with zero failure at y_0 .**
 - $y_0 = 5000$, $x_0 = 55.3$, $t_0 = 80$
 - $y_0 = 7500$, $x_0 = 51.7$, $t_0 = 90$
 - $y_0 = 10000$, $x_0 = 62.3$, $t_0 = 50$
 - etc...
- **There are many alternatives.**

General formulation

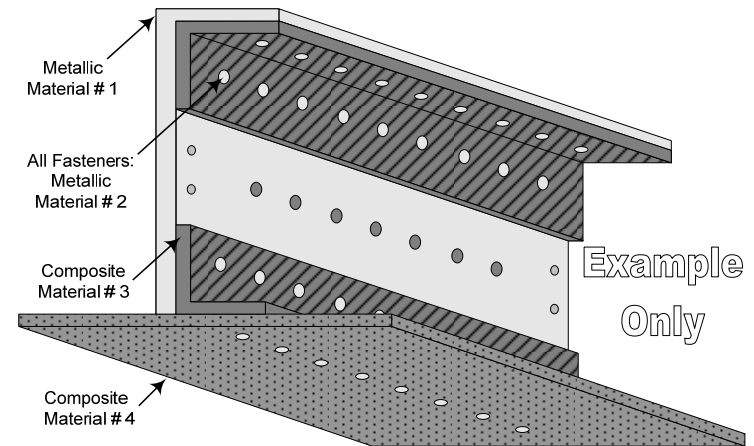
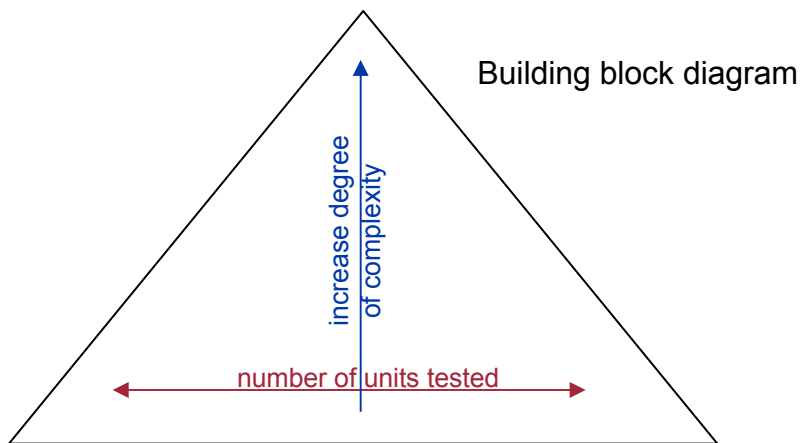
- **Binomial probability of k out of n failure to demonstrate R_0 reliability with C_0 confidence**
- **Reliability function in relation with test variables x**
- **For $n_d < n$, treat the reliability relationship (life time regression model) as the best guess of the phenomenon.**
- **Given some design / certification requirements (fixing some of life and test variables), determine the other test variables (x_d) and a sample size (n_d) that would satisfy the original requirements.**
- **Setting $k=0$ is a common practice.**

Remarks

- **RDT appears in the literature since 1970's primarily associated with the automotive industry. Aerospace industry has a similar approach called load enhancement factor. However, they are distinct enough to be treated differently.**
- **RDT pros:**
 - **obvious benefits in reducing cost and development time**
 - **flexibility in re-designing a test plan**
 - **allow engineer to consider the impact of changing the test conditions**
- **RDT cons:**
 - **assume the relationship is true and fix; no account for sampling variation from the historical knowledge**
 - **typical concern of altering failure mode when product is stressed beyond its normal operating condition**
 - **“get away with murder”; testing one unit is good enough ?**

Remarks

- **Scale up testing:**
 - **RDT and its philosophy is used to build knowledge on a scale up test. Many certification tests cannot test large number of complex part. Complex components have multiple simple parts. Most fatigue tests on complex components are not done on constant stress load.**



*Detail does not represent any particular aircraft structure configuration and is used for illustration purposes only.

- **No guarantee success; RDT is a strategy to set up a test plan.**