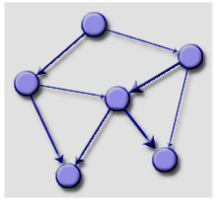

NIPS 2006 Workshop
Causality and Feature selection

Discovery of linear acyclic models in the presence of latent classes using ICA mixtures

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Statistical causal inference for empirical research

- An effective way is to conduct an experiment with random assignment (Holland 1989, Rubin 1974)
- Many situations where it is costly or esthetically difficult to conduct experiments
- Need to develop useful methods to find likely causal models from non-experimental data
- We have proposed a new method for continuous-valued data using independent component analysis called LiNGAM
- We extend the method to cases where latent classes are present (a nonlinear extension)

Independent Component Analysis

(Comon 1994; Hyvarinen et al., 2001)

$$\mathbf{x} = \mathbf{A}\mathbf{s}$$

- A variant of factor analysis
- A is unknown. Typically, A is square
- s_i (continuous variables) are assumed to be non-Gaussian and independent
- Estimable including the rotation (Comon, 1994)

Linear acyclic models

(Bollen, 1989; Pearl, 2000; Spirtes et al., 2000)

- Continuous variables
- Assumptions
  1. Directed acyclic graph (DAG)
  2. Linearity
  3. Disturbances (errors) are independent (no hidden confounders)

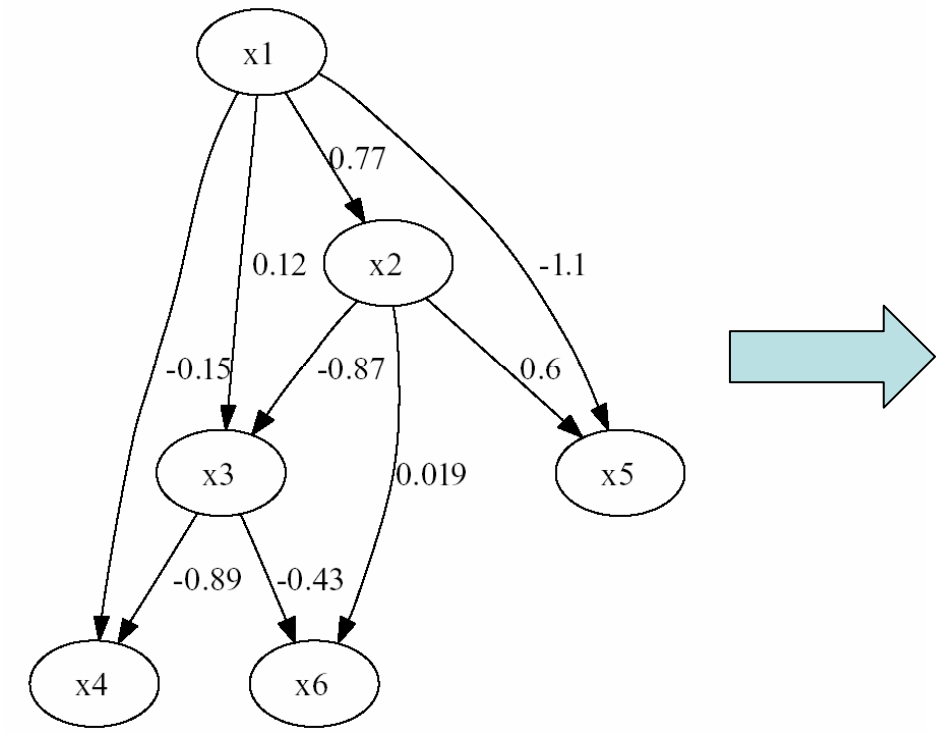
$$x_i = \sum_{k(i)>k(j)} b_{ij} x_j + e_i \quad \text{or} \quad \mathbf{x} = \mathbf{B}\mathbf{x} + \mathbf{e}$$

Our goal

- We know
  - Data X is generated by $\mathbf{x} = \mathbf{B}\mathbf{x} + \mathbf{e}$
- We do NOT know
  - Connection strengths: B
  - Order: k(i)
  - Disturbances: ei
- What we observe is data X only

- **Goal**
  - Estimate B and k using data X only!

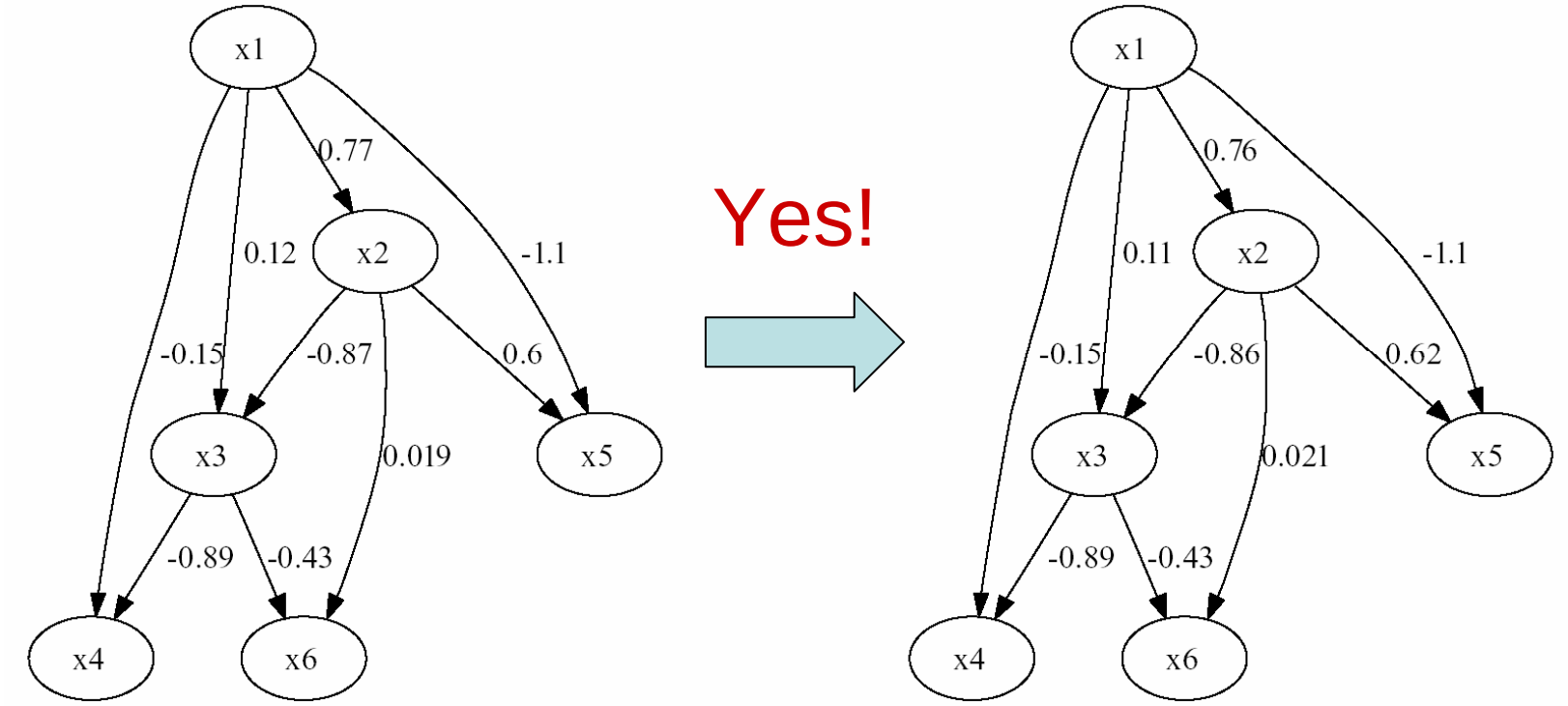
Can we recover the original network?



Original network

?

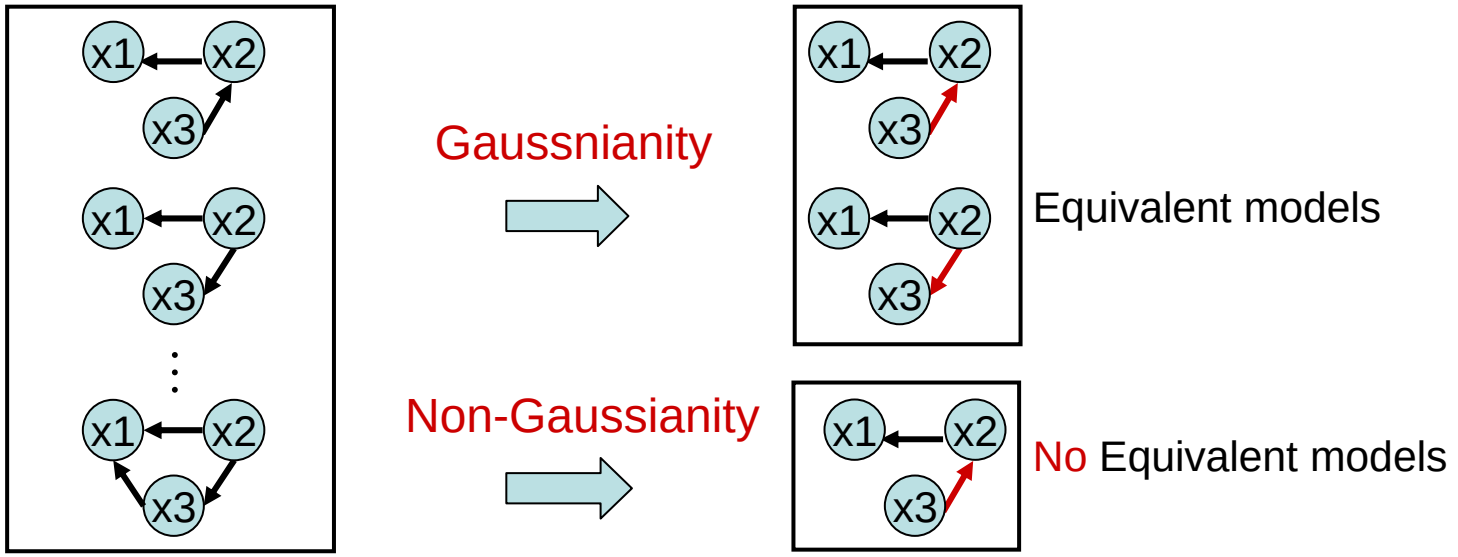
Can we recover the original network using ICA?



Original network **Estimated network**

Discovery of linear acyclic models from non-experimental data

- Existing methods (Bollen,1989; Pearl, 2000; Spirtes et al., 2000)
  - Gaussian assumption on disturbances ei
  - Produce many equivalent models
- Our LiNGAM approach (Shimizu et al, UAI2005, 2006 JMLR2)
  - Replace Gaussian assumption by non-Gaussian assumption
  - Can identify the connection strengths and structure



A simple example

- Two regression models with opposite directions of arrows

- Linear-Gaussianity
  - Provide the same covariance structure
  - No way to distinguish between the two models

- Linear-Non-Gaussianity
  - Can evaluate model fit in terms of second- and fourth-order moments (Shimizu & Kano, 2006, JSPI)

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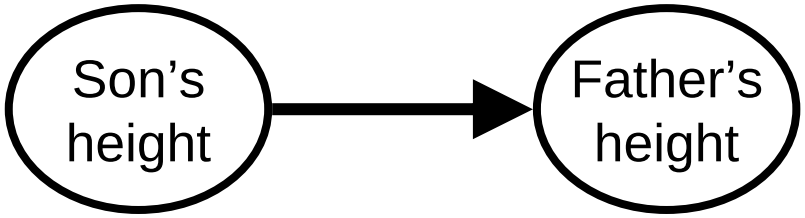
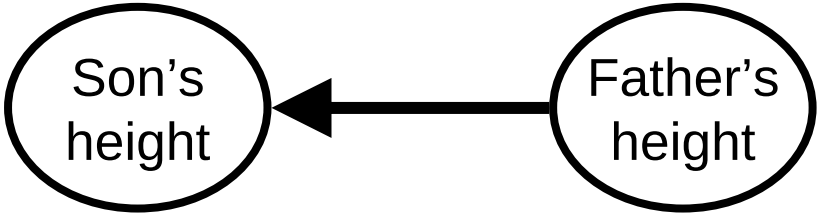
p value = 0.04



p value = 0.53



126 samples

Linear Non-Gaussian Acyclic Models

(Shimizu et al, 2006 JMLR)

- As usual, linear acyclic models,
  but disturbances e_i are assumed to be non-gaussian:

$$x_i = \sum_{k(i)>k(j)} b_{ij} x_j + e_i \quad \text{or} \quad \mathbf{x} = \mathbf{B}\mathbf{x} + \mathbf{e}$$

- Note that B can be permuted to be strictly lower triangular if order k is known due to the DAG assumption (Bollen, 1989)
- No `faithfulness' or `stability'

Basic insight

- Observed variables x_i are linear combinations of non-gaussian independent disturbances e_i

$$\mathbf{x} = \mathbf{B}\mathbf{x} + \mathbf{e}$$

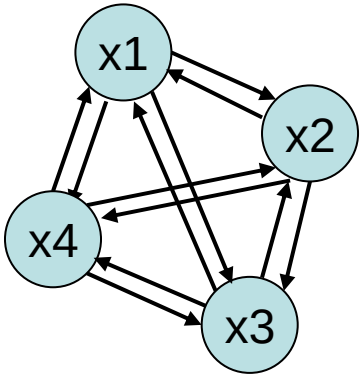
$$\Rightarrow \quad \mathbf{x} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{e}$$

$$= \mathbf{A}\mathbf{e}$$

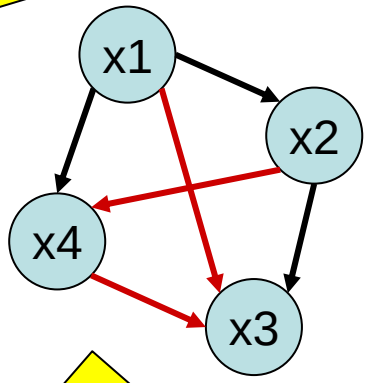
- Hence, we have a classic case of ICA

Outline of LiNGAM algorithm

1. Estimate B by ICA
+ post-processing

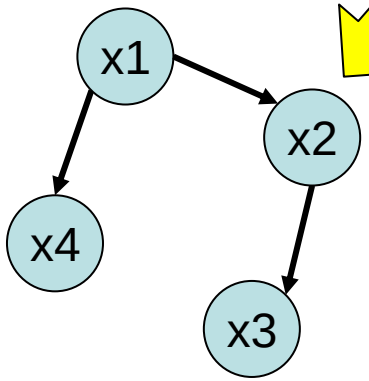


2. Find an order k(i) (DAG)



3. Prune redundant edges

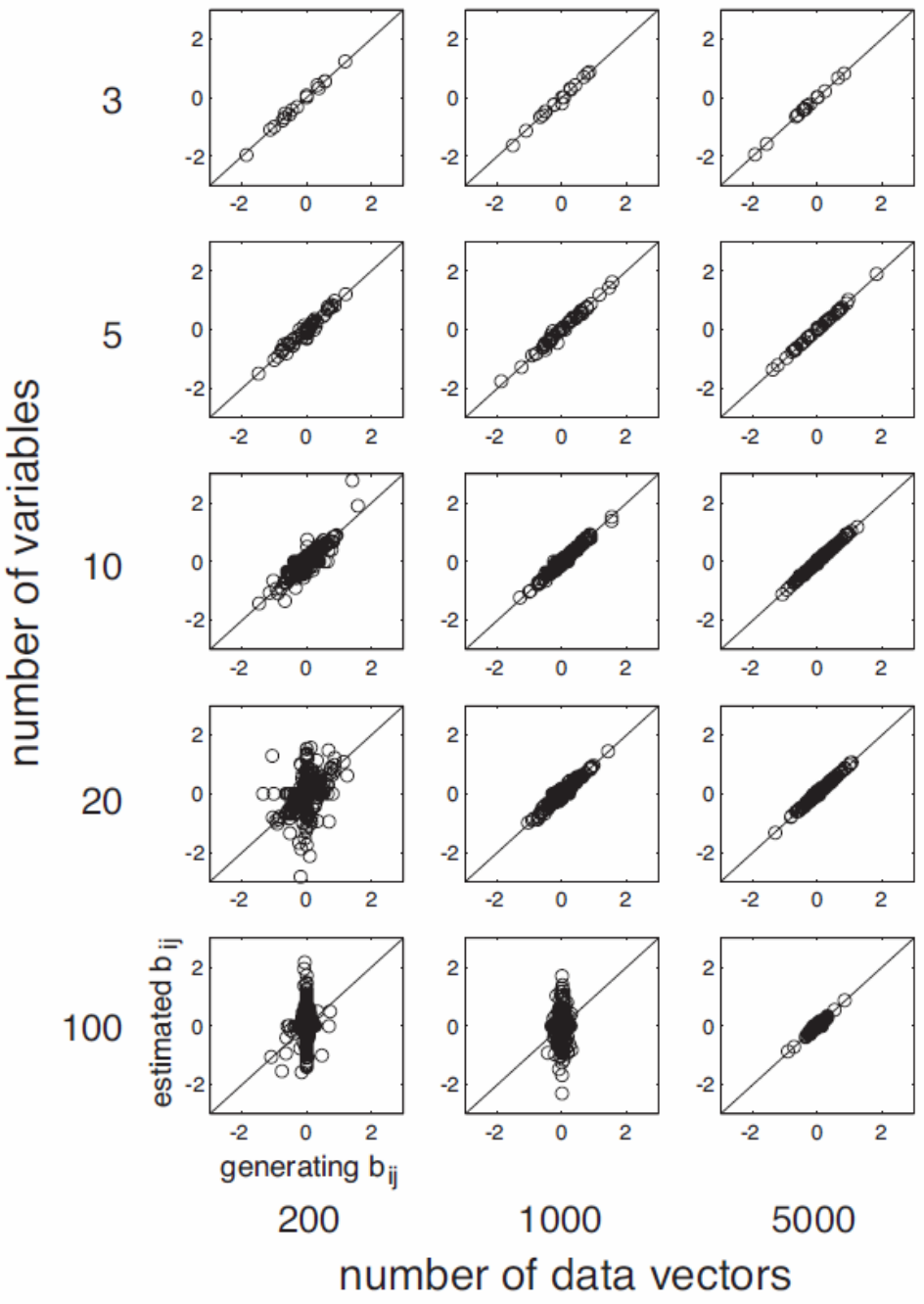
Wald test, Bootstrapping+OLS, etc.



Simulations

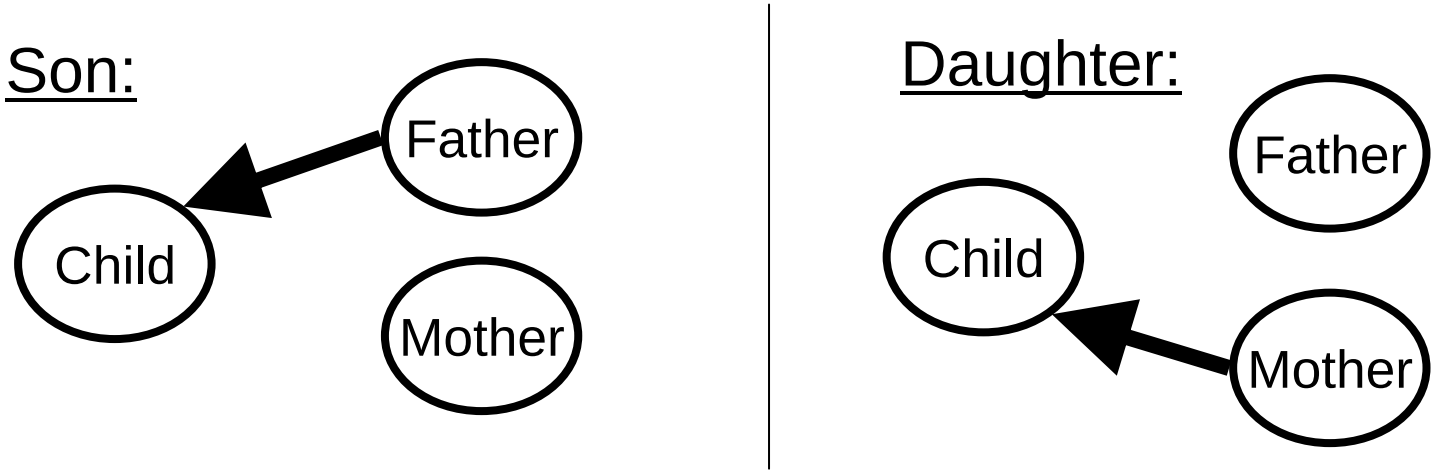
- Randomly generated B
- Various non-gaussian disturbances

- More along the main diagonals
  → more accurate estimation



An extension to cases where latent classes are present

- Hereditary effect on height (Kimura, 1974)



- Different network structures between different classes
- Often difficult to find class-membership
- Need a method to estimate in a data-driven way
  - latent classes of samples that are similar
  - the number of classes

Model estimation by ICA mixtures

- LiNGAM model for each class q

$$\mathbf{x} = \mathbf{B}_q \mathbf{x} + (\mathbf{I} - \mathbf{B}_q)\boldsymbol{\mu}_q + \mathbf{e}_q$$

Mean vector of x

- Estimable by ICA mixtures (Lee et al, 2000)
  - Min. Beta-divergence method (Mollah et al, 2006) learns A_q as well as the number of latent classes

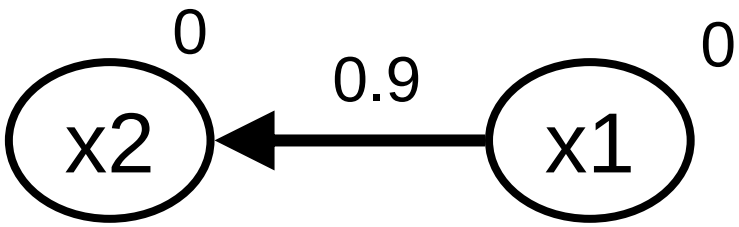
$$p(\mathbf{x} \mid \Theta) = \sum_{q=1}^{Q} p_q(\mathbf{x} \mid \boldsymbol{\mu}_q, \mathbf{A}_q) p(C = q),$$

$$\textbf{where } \mathbf{A}_q = (\mathbf{I} - \mathbf{B}_q)^{-1}$$

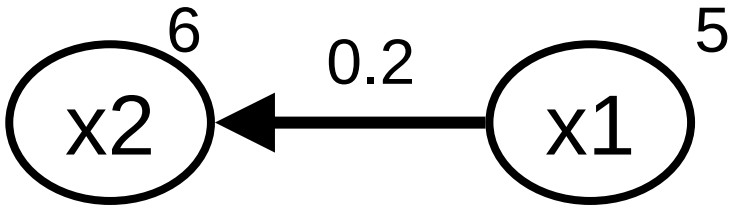
Illustrative example

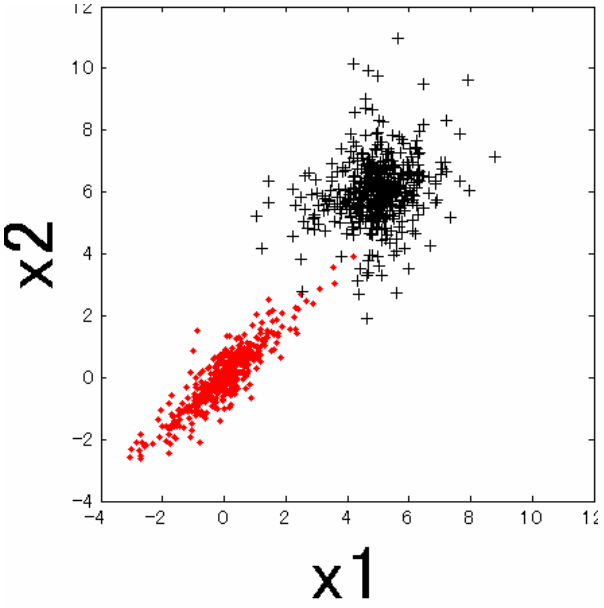
- Two classes have
  - the same causal order x1→x2
  - Different connection strengths between the classes (interaction effects)
  - Different means of x1 and x2
- Laplace distribution

Class 1:



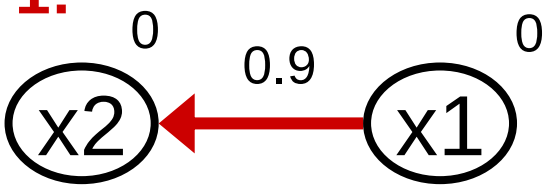
Class 2:





True:

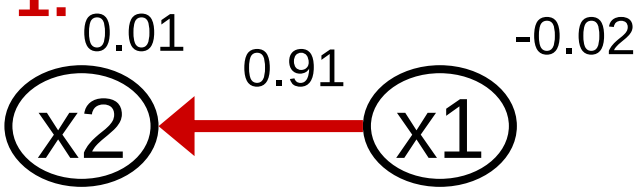
Class 1:



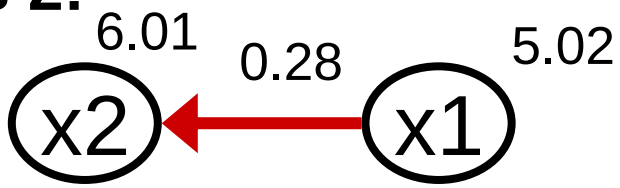
Class 2:

Latent class LiNGAM:

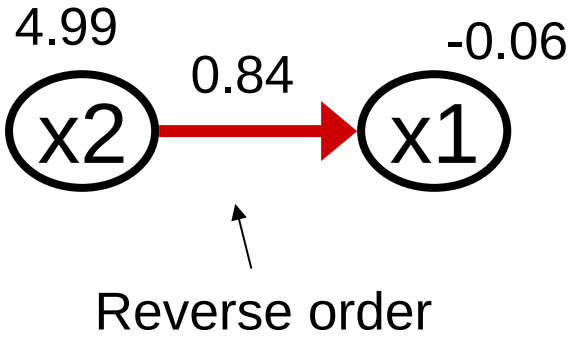
Class 1:



Class 2:



Regular LiNGAM:



Conclusions

- Discovery of linear acyclic models from non-experimental data is an important topic of current research
- A common assumption is linear-Gaussianity, but this leads to a number of indistinguishable models
- A non-Gaussian assumption allows all the connection strengths and structure of linear acyclic models to be identified (LiNGAM algorithm)
- In this contribution, we extended the method to the cases where latent classes are present (a nonlinear extension)
- Matlab/Octave code of the regular LiNGAM: http://www.cs.helsinki.fi/group/neuroinf/lingam/